

# 1 Lionel W. McKenzie

Tapan Mitra and Kazuo Nishimura

## 1.1 Preamble

Lionel McKenzie is one of the giants of twentieth-century neoclassical economics. The most productive period of his research coincided with the twenty-five years of the postwar period, which saw the major themes of neoclassical economics crystallized, and which also saw the development and use of more fundamental mathematical methods toward this end. He contributed greatly to both facets of this research program and, indeed, helped to shape the direction in which it developed.

To us, his students at the University of Rochester in the seventies, there is another dimension to Lionel McKenzie. We had the privilege of learning from him firsthand much of what we know about how to approach economic theory. The learning experience from his classroom lectures was supplemented by the handwritten lecture notes he would circulate and followed by presentations by students on a set of papers (typically unpublished current work by researchers from all over the world) selected by him. For us, that entire experience was to have permanent effects. The definitive books by Debreu (1959), Nikaido (1968), and Arrow and Hahn (1971), dealing with roughly the same subject matter, were illuminating on a variety of issues, but were *different* in their approaches from McKenzie's. The way we understand the subject matter even today was decided years back by the way we first understood some of the central concepts, the way McKenzie introduced them to us.

We wished for the lecture notes to be published in book form someday, so our experience could be at least partially shared by others in the profession. We felt encouraged when, upon his retirement, Lionel McKenzie began organizing his lecture notes for publication. And, our

hope was realized when *Classical General Equilibrium Theory* was published by the MIT Press in 2002.

When the book was in preparation, we had the benefit of looking at early drafts of the manuscript. It occurred to us at that point that it would be ideal to publish a companion volume, consisting of a selection of Lionel McKenzie's papers. The idea, met with immediate approval from Lionel McKenzie himself, comes to fruition with this book.

It is often desirable in such collections to provide the reader with an overview of the selected papers, a guide which could be used to explore some papers in greater depth than others, depending on one's tastes. But this task has already been accomplished by our teacher. In a paper which he contributed to a special issue of the *Nagasaki Prefectural University Review* in memory of Professor Yasuo Uekawa, McKenzie reviewed his own contributions to equilibrium, trade, and capital accumulation. A slightly revised version of the paper was published in the *Japanese Economic Review*, and it provides the perfect lead article of the present collection.

The purpose of this chapter is therefore somewhat different. We provide here a sketch of McKenzie's academic career with the hope that it will help illuminate his contributions to the areas of equilibrium, trade, and growth, by providing us with the relevant backdrop as well as hints about possible influences. The chapter relies on McKenzie's own recent retrospective, "A Scholar's Progress," his reminiscences in "The First Conferences on the Theory of Economic Growth," as well as on Roy Weintraub's account of the development of the theory of competitive equilibrium in his *Journal of Economic Literature* article (1983) and book (1985). However, it also relies on our own reading of McKenzie's papers.

## 1.2 A Life in Academia

### 1.2.1 In Search of a Specialization

Lionel McKenzie was born in 1919 in Montezuma, Georgia. His mother, in her youth a teacher, encouraged him to read the Harvard Classics. He attributes his introduction to economics to a reading of Adam Smith's *Wealth of Nations* one summer, while he attended Middle Georgia College in Cochran.

After graduating from junior college, he transferred to Duke University to enter an honors course in philosophy, politics, and economics.

The Great Depression of the 1930s was probably responsible for his choosing to concentrate in economics. In his final year at Duke, he won a Rhodes scholarship to pursue the same course at Oxford, but his plan was postponed indefinitely by the Second World War.

In 1939, he entered the Princeton Graduate College to work toward a Ph.D. in economics. There, he was greatly influenced by Frank Graham, who introduced him to multisector, many-country models of international trade, and by Oskar Morgenstern, whose course on economic theory included a critical study of *Value and Capital* by John Hicks. In 1941, with the United States at war, McKenzie left Princeton and spent a year with the Office of Civilian Supply in the War Production Board, and then in the Navy up to 1945.

In 1946, he obtained an appointment as an instructor at MIT. It appears that, even though he was familiar with Samuelson's work, his stay at MIT had no lasting influence on his academic interests, and after a couple of semesters there he resigned to take up his deferred scholarship at Oxford.

At Oxford, he was entered in the D.Phil. program, where his supervisor was John Hicks. While at Oxford, he spent a significant amount of his time studying philosophy and did not pursue the mathematical approach to economics. He embarked on a research project on modern welfare economics and wrote a draft of a thesis on the subject. His examiners (Roy Harrod and Hibert Henderson) ruled that it was not a finished product and would need to be revised. When he did not agree to do this, McKenzie had to settle for a B.Litt. degree.

Upon returning to the United States in 1948, he joined Duke University as an assistant professor. There, he wrote his paper "Ideal Output and the Interdependence of Firms," published in the *Economic Journal* in 1951. McKenzie refers to this work as an indirect outcome of his research project at Oxford, although it was not included in his thesis. At Duke, he became acquainted with Tjalling Koopmans's work on activity analysis. The theory of activity analysis immediately appealed to McKenzie as providing the sort of method that he could have used in his paper on ideal output. But, it also must have had a more profound influence, for in 1950, after teaching at Duke for two years, he applied to Jacob Marschak to visit the Cowles Commission in Chicago and became a University of Chicago graduate student in economics, a decisive turn in his academic career.

### 1.2.2 The Year at the Cowles Commission

At the Cowles Commission, McKenzie devoted considerable time to learning mathematics, taking courses from Paul Halmos, Irving Kaplansky, Saunders MacLane, and Jimmy Savage. He attended the classes of Tjalling Koopmans on activity analysis and econometrics and of Jacob Marschak on decision making under uncertainty. His colleagues in these courses included Martin Beckmann, John Chipman, and Edmond Malinvaud. Karl Brunner, Gerard Debreu, Leonid Hurwicz, and Harry Markowitz were also in the Cowles Commission group at that time.

In Koopmans's class, McKenzie wrote a term paper entitled "Specialization in Graham's Model of World Trade," a revised version of which appeared in 1954 in the *Review of Economic Studies* under the title "Specialization and Efficiency in World Production." Koopmans was pleased with the term paper and suggested that McKenzie work on the factor price equalization theorem of Samuelson in an activity analysis model. But McKenzie did not follow up on this suggestion until several years later.

On the walk back from a class by Koopmans on the relation between activity analysis and competitive equilibrium, McKenzie asked Koopmans about the issue of existence of a competitive equilibrium. Koopmans indicated that the existence question had not yet been answered. While at the Cowles Commission, McKenzie began thinking seriously about the question, especially in the context of Graham's models of international trade. It is perhaps significant (in view of developments that followed over the next year) that during his year in Chicago, McKenzie acquired a set of notes from a seminar on convex sets given by Marston Morse at Princeton, as well as a Cowles Commission paper by Morton Slater on Kakutani's fixed point theorem.

### 1.2.3 On the Existence of Competitive Equilibrium

Upon returning to Duke, McKenzie studied the papers by Abraham Wald (1951) and John von Neumann (1945) on the existence of equilibrium, and proceeded to write his own paper "On Equilibrium in Graham's Model of World Trade and Other Competitive Systems," which he presented at the Chicago meeting of the Econometric Society in December 1952. The Arrow-Debreu paper on the existence of a competitive equilibrium was presented at the same meeting, and both were later published in *Econometrica* in 1954. The approaches taken in the two papers on this central problem of general equilibrium theory were

quite different. The Arrow-Debreu paper, which viewed competitive equilibrium as the outcome of a game, was directly influenced by the paper of Nash (1950) on non-cooperative games, and by the generalization of a Nash equilibrium to a social equilibrium by Debreu (1952). McKenzie's paper was inspired by the papers of Wald (1951) and the theory of activity analysis developed by Koopmans (1951).

According to McKenzie, while writing his term paper on specialization at the Cowles Commission, the idea came to him of mapping the social demand from the origin on the world production possibility frontier as a possible device for proving the existence of competitive equilibrium. As his 1954 paper showed, this device, when combined with the price support property of efficient points, makes it possible to view a competitive equilibrium as a fixed point of an upper semicontinuous correspondence from the price simplex to subsets of that simplex. The map itself suggests use of the Kakutani fixed point theorem (1941) as a natural method of establishing the existence of a competitive equilibrium.<sup>1</sup>

The map constructed by McKenzie is a composition of three maps, and it is possible to explain in geometric terms what each component map does. In fact, McKenzie has indicated that the notes on convex sets by Marston Morse allowed him to understand the existence problem geometrically. Moving somewhat ahead of the story, it is interesting to note that when Dorfman, Samuelson, and Solow published their classic *Linear Programming and Economic Analysis* in 1958, they chose to present the proof of existence of competitive equilibrium very much along the lines of McKenzie's 1954 paper.

### 1.2.4 Research at Duke

As already noted, Koopmans had suggested McKenzie work on the factor price equalization theorem, which is where McKenzie turned his attention next; he showed that if there is a choice of factor prices that put each country's factor supply in the interior of the diversification cone, then those factor prices would have to prevail in all countries in a world equilibrium under free trade. This result was published in *Econometrica* in 1955 and still remains the state-of-the-art result on the subject.

McKenzie then continued his research on the existence of competitive equilibrium; while his earlier contribution had started with demand functions, his new approach focused on consumer preferences as the primitive objects of study. This led him to give a simple approach

to demand theory starting from assumptions on consumer preferences, in the paper “Demand Theory without a Utility Index,” published in the *Review of Economic Studies* in 1957. This elegant paper forms the basis of demand theory, and a version of it is now covered in every standard graduate text on microeconomic theory.

His research on preference orderings led him to introduce preferences of consumers that depend on what other consumers buy. His paper on the existence of equilibrium with such *dependent* consumer preferences was presented at a Symposium on Linear Programming and published in its proceedings in 1956. At this symposium, Oskar Morgenstern indicated that he would make a proposal to the Princeton economics department to grant McKenzie a Ph.D. So, McKenzie obtained his degree by putting together the papers he had written and submitting them to Princeton.

It was around this time that McKenzie completed his definitive work on existence of competitive equilibrium, weakening the assumptions made by Arrow-Debreu (1954) and by himself in his earlier papers on the subject in 1954 and 1956. This was accomplished by introducing the notion of an irreducible economy, following a suggestion from David Gale. This paper was later published in *Econometrica* in 1959.

In the spring semester of 1956, while still at Duke, McKenzie visited the Cowles Foundation at Yale University. There he worked on the Ramsey theory of optimal taxation, and on a study of matrices with dominant diagonals, with applications to economics. The research on the Ramsey theory of optimal taxation was reported at the summer meeting of the Econometric Society in Ann Arbor, Michigan, but never submitted for publication. The study on dominant diagonal matrices later became McKenzie’s contribution to the conference on Mathematical Methods in the Social Sciences held at Stanford in 1959. It showed the wide range of economic problems to which the concept of a dominant diagonal matrix could be applied fruitfully.

### 1.2.5 Move to Rochester

During his visit to the Cowles Foundation in 1956, McKenzie was approached by the University of Rochester to head an economics department there, with the aim of developing a Ph.D. program. He accepted the offer and moved from Duke to Rochester in the fall of 1957. During his first academic year, McKenzie recruited Ronald Jones, and the program’s first Ph.D. student in economics, Akira Takayama, was admitted in that year. Richard Rosett and Edward Zabel joined the

faculty the following year, and a full-fledged doctoral program was put into place. Some of the first students to join that program were Akihiro Amano, Hiroshi Atsumi, and Yasuo Uekawa. The graduate program grew in strength and reputation over the years, and the faculty expanded rapidly under McKenzie's chairmanship from 1957 to 1966. In 1995, when the National Research Council published its definitive study ranking departments of universities in the United States in the various disciplines, the economics department of the University of Rochester ranked among the top ten, as judged by the quality of its graduate program.

### 1.2.6 Final State Turnpike Theory

In 1958 Dorfman, Samuelson, and Solow (DOSSO) had published *Linear Programming and Economic Analysis*. The book analyzed a planning problem in which the objective was to maximize terminal stocks of capital goods (in certain prespecified proportions) over a long but finite horizon. It was indicated in DOSSO that plans that were optimal with respect to such an objective should stay, for most of the planning horizon, near the von Neumann equilibrium,<sup>2</sup> representing a balanced growth path with the maximum rate of growth. Drawing its analogy from the effectiveness of using the turnpike while driving over long distances, it came to be known as the *turnpike conjecture*.<sup>3</sup> McKenzie read the book very soon after moving from Duke to Rochester and spent the next twenty-five years exploring various aspects of the turnpike conjecture and establishing turnpike theorems that have greatly enriched the literature on economic dynamics.

In 1960 McKenzie, Morishima, and Radner independently provided the first formal proofs of the (final-state) turnpike theorem. It appears that all of them were influenced to some extent by John Hicks, who gave seminars at various universities around this time, reporting on the difficulty of formally establishing a turnpike theorem (see also, in this connection, the paper by Hicks (1961)). McKenzie (1963) and Morishima (1961) used Leontief models of production in their papers, but their approaches were very different from each other. Radner (1961) modeled the production set as a convex cone, which had become the standard generalization of the von Neumann model, but his assumption that the input-output combinations on the von Neumann ray were the only profit-maximizing combinations at the von Neumann prices made the technology quite restrictive. Radner's method of proof was elegant and it conveyed the sense of the original turnpike conjecture; it

was to influence almost all of McKenzie's future writings on turnpike theory.

McKenzie quickly recognized that Radner's technique of proof was ideally suited for proving turnpike theorems for more general models of capital accumulation. He applied it to the dynamic Leontief model<sup>4</sup> and to the von Neumann model,<sup>5</sup> both of which are examples of well-known models in which Radner's "unique profitability condition" fails to hold. McKenzie showed that Radner's method can be used to establish the very general theorem that paths optimal with respect to the final state would have to lie close to a facet of the technology set (containing the von Neumann ray) consisting of all those activities that maximize profits at the von Neumann equilibrium prices.

These results might suggest that proximity of optimal paths to the von Neumann ray itself was not to be obtained in general. However, further investigations by McKenzie<sup>6</sup> indicated that paths on the facet have an additional stability property. This ensured the proximity of optimal paths to a subset of the facet activities, a subset that degenerated to the von Neumann ray in many important cases. The definitive account of this theory was presented by McKenzie in 1968, using the theory of matrix pencils developed by Gantmacher.<sup>7</sup>

### 1.2.7 The Uniqueness Debate

As an interlude to these exciting developments of turnpike theory, McKenzie wrote two pieces on the inversion of cost functions. To place them in the proper context requires going back more than a decade to a paper by Samuelson in 1953 in which, generalizing his own two-by-two-by-two factor price equalization result, a condition on the Jacobian matrix of the cost functions was conjectured by him to be sufficient for inversion of cost functions and therefore for factor price equalization (in the case of non-joint production) under "incomplete specialization." As we have already indicated, the most general result on factor price equalization (in a model that included cases of joint production and nonsmooth production relations, and allowed for specialization) was established by McKenzie in 1955. However, Samuelson's conjecture remained an issue of interest for the inversion of cost functions and for the uniqueness of competitive equilibria arising in contexts other than in international trade. Nikaido<sup>8</sup> gave a counterexample to Samuelson's conjecture in the context of global univalence of solutions to a system of nonlinear equations in general, and subsequently Gale and Nikaido



(1965) established that a stronger condition than Samuelson's was sufficient.<sup>9</sup> But, Nikaido's counterexample itself lacked the restrictions that cost functions would impose on the relevant equation system.

Pearce and James (1952) had rightly observed that if the Jacobian of the cost function were nonsingular, it would allow a local inversion of the cost function, but this condition was not guaranteed to ensure global inversion. However, Pearce (1959) had made the conjecture that the nonvanishing Jacobian condition might be sufficient after all for global invertibility of cost functions.

McKenzie set the matter to rest in 1967 by providing a definitive counterexample to the global invertibility of cost functions under the nonvanishing Jacobian condition.<sup>10</sup> This counterexample established the important result that determinateness of equilibrium was not to be expected as a generic outcome. It is an important instance of the sheer power of mathematical reasoning. Under the nonvanishing Jacobian condition, what one can see from the inverse function theorem is that the cost function must be locally invertible. But, the condition is clearly weaker than the Gale-Nikaido conditions, so whether global inversion holds or not is not easy to guess. The matter must be settled with a theorem or a counterexample, where the mathematics plays the important role, and economic intuition can provide only limited assistance.

In 1970 Debreu showed, using Sard's theorem, that the generic outcome of equilibrium systems was a *finite number* of equilibria, thereby providing a formal justification for *local* comparative static exercises. Following his influential paper, mathematical economists conducted a considerable amount of research on uniqueness of solutions to systems of nonlinear equations, defined by *smooth functions*, with the hope of producing stronger results on uniqueness of equilibria arising in economics by exploiting the methods of differential topology. However, McKenzie's counterexample clearly defined the limits to which this research program could be pushed.<sup>11</sup>

### 1.2.8 On the Ramsey Turnpike

Apart from this detour, McKenzie's research remained firmly focused on turnpike theory. While growth of capital stocks might be a reasonable objective in the initial stages of modernization for planned economies, researchers recognized that the motivation for studying such an objective was somewhat forced. Consumer preferences were ignored in planning exercises over such horizons, but the more basic objection

was that if growth of capital stocks were indeed a reasonable goal for planned economies in the initial phases of their development, that should be the *consequence* of attaining a more fundamental objective rather than being imposed as an *objective* itself.

Such a fundamental objective should reflect consumer preferences and ideally should extend over an indefinite future, since specifying some fixed terminal date would itself be an ad hoc imposition on the problem.<sup>12</sup> This brought the profession's interest to focus precisely on Ramsey's classic 1928 paper on optimal savings.<sup>13</sup>

McKenzie was a leading contributor in establishing this shift of focus, which principally was accomplished in two conferences on optimal economic growth. The first was at the University of Rochester, led by McKenzie; the second was at Stanford University, led by Kenneth Arrow. Several papers initially presented at these conferences later were published in a *Review of Economic Studies* symposium in 1967.

The reformulation of the objective function along the lines of Ramsey meant that utility at each point in time would be derived from consumption and leisure choices, and future utilities would be treated as current ones in the planner's objective function, since Ramsey considered discounting of the future utilities to be "ethically indefensible." This brought to the forefront the question of existence of optimal paths over an infinite horizon, which had not been a major issue in the final-state turnpike literature. On the other hand, the similarity of the two theories derived from the similarity of the methods used to establish asymptotic properties of optimal paths. While the papers of Gale and McFadden,<sup>14</sup> and later Brock,<sup>15</sup> primarily were concerned with the first topic, Atsumi<sup>16</sup> and McKenzie recognized that Radner's "value-loss lemma" could be adapted to the new setting to obtain turnpike theorems for both finite- and infinite-horizon optimal paths when future utilities were undiscounted, with the golden-rule equilibrium (associated with a program yielding maximum sustainable utility) replacing the notion of the von Neumann equilibrium. McKenzie's comprehensive treatment of this topic was his contribution to the Hicks *festschrift* in 1968.

In light of later developments in this area of research, it is worth mentioning at this point that in the first conference on optimal growth (mentioned above), McKenzie introduced the formulation of utility derived at each point in time, which has come to be known as the "reduced form" version of the optimal growth model. In this formula-

tion, utility is derived from beginning- and end-of-period stocks of goods.<sup>17</sup> Consumption, leisure, and stock effects on returns all can be captured in this formulation, but these “primitive” variables determining utility do not appear explicitly in the reduced form version. The formulation has the advantage over other formulations in providing the essential mathematical form of the *intertemporal* problem, where the *atemporal* problem already has been “solved” in arriving at the reduced form utility function. This has become the standard formulation of the utility function used in optimal growth theory.

### 1.2.9 Discounting and Long-Run Behavior

Along with a shift in emphasis of many economies away from planning at the national level was a corresponding change in interpretation of dynamic optimization problems of the Ramsey type. The problem being solved was previously viewed as a normative problem that the “social planner” ought to solve but came to be viewed as a descriptive problem a typical representative agent (more precisely, an infinitely lived dynasty of the typical agent) solves. The Ramsey objection to discounting future utilities as “ethically indefensible” on the part of the social planner was no longer relevant. If the representative agent did discount the future, the optimization problem would have to reflect this. Thus, the central problem to be solved in describing the agent’s behavior would be a discounted dynamic optimization problem of the Ramsey type.

This reformulation of the focus of the subject had two significant consequences. The issue of the existence of an optimal program, which had occupied center stage for undiscounted dynamic optimization models, became a relatively unimportant aspect of the theory for discounted models because it was a relatively straightforward exercise, under discounting, to establish the existence of an optimal program. In contrast, description of dynamic behavior of optimal programs became considerably more difficult.

Examples due to Kurz (1968) for continuous-time models and Sutherland (1970) for discrete-time models indicated that multiple stationary optimal states could exist. Further, even if the stationary optimal state were unique, optimal programs starting from other initial states need not converge to it over time. Then, Samuelson (1973) presented an example due to Weitzman, which showed that optimal programs could cycle around a unique stationary state independent of the

magnitude of the discount factor, and these cycles were not “boundary phenomena.” While this destroyed any hope of a general turnpike theorem for discounted models, Samuelson conjectured that with (differential) strict concavity of the utility function, a turnpike property for optimal programs would continue to hold for high discount factors. This led to a considerable literature on the discounted turnpike problem. In alternative frameworks, Samuelson’s conjecture was shown to be valid by Brock and Scheinkman (1976), Cass and Shell (1976), Rockafellar (1976), and Scheinkman (1976).

McKenzie was very much part of these developments. The Mathematical Social Sciences Board Conference in Squam Lake, where these papers (among others) were presented, received his encouragement as well as close scrutiny, as was quite common with McKenzie’s oral presentations. A more concrete evidence of his contribution can be found in the proof of the turnpike theorem presented by Brock and Scheinkman (1978) in a discrete-time setting.

McKenzie also remained somewhat distant from this line of research, though. He was on a quest for the general result of the subject, and the theory of global asymptotic stability of optimal paths when the discount factor is sufficiently close to 1 had not come to terms with the Weitzman-Samuelson example. In 1980–1981, McKenzie spent a year away from Rochester as Taussig Research Professor at Harvard University. There, he wrote his paper on the “neighborhood turnpike theorem” when future utilities are discounted. This paper was published in the *Journal of Economic Theory* in 1982, and its generalization (to the case where no strictness in concavity of the utility function is assumed) was published in the same journal the following year. They represent a full account of what rightly can be regarded as the general result of the subject.

When the discount factor is close to 1, a stationary optimal stock is close to the golden-rule stock (of maximum sustainable utility). McKenzie had always regarded Jose Scheinkman’s (1976) “visit lemma” as a basic result in the discounted case. This said that an optimal path must visit any preassigned small neighborhood of the golden-rule stock at least once, when the discount factor is sufficiently close to 1. This is, of course, different from the typical global asymptotic stability result. The latter result (when the discount factor is close to 1) can be obtained by ensuring that the “basin of attraction” of the stationary optimal stock (which clearly exists when the discount factor equals 1) does not vanish for discount factors close to 1.

The last requirement is fairly strong; it is a kind of uniform local asymptotic stability condition with respect to the discount factor. In particular, it rules out cyclical paths, and therefore it does not accommodate examples (of the Weitzman-Samuelson type) where cycles persist for all discount factors, no matter how close to 1. Now, if the only cycles that persist are cycles of small amplitude around the turnpike, the turnpike still would be a good approximation to long-run optimal behavior. So, McKenzie's idea was to use the "visit lemma" to make optimal paths visit a neighborhood of the golden-rule stock and then trap them in a (possibly somewhat larger) neighborhood of the stationary optimal stock by showing that the stationary optimal stock was stable in the sense of Lyapounov. This allows optimal paths to cycle or even exhibit more complicated behavior while being confined to this neighborhood.

### 1.2.10 A Presidential Lecture

In 1977, Lionel McKenzie was elected President of the Econometric Society. For his presidential lecture, he chose the topic of existence of competitive equilibrium, addressing one of the most severe shortcomings of general equilibrium theory up to that time. This was the issue of ensuring survival of agents in equilibrium, without assuming that agents can survive on their own.

In their proofs of existence of a competitive equilibrium, Arrow and Debreu (1954) and McKenzie (1956) had maintained the assumption that agents can survive on their own. Koopmans (1957), evaluating the contributions made to general equilibrium, had regarded this area as one needing additional research, remarking, "In modern society few of us can indeed survive without engaging in exchange." McKenzie (1959) had maintained this assumption in slightly modified form and had changed it to the more direct version used in Arrow-Debreu when in 1961 he published his corrections to the 1959 contribution. Arrow and Hahn (1971) had introduced the alternative notion of resource relatedness, but this implied in a very strong sense that agents could survive without trading.

The assumption that individuals can survive without trading was needed to ensure continuity of demand functions, as McKenzie had recognized even in his 1954 paper, where demand functions were primitives, and not derived from assumptions of consumer preferences on their consumption sets. Jim Moore (1975) was able to overcome this problem by using the method of Negishi instead of dealing with

properties of demand functions. He showed that the assumption that individuals can survive can be dropped if the economy were assumed to be irreducible in the sense of McKenzie (1959, 1961).

McKenzie (1981) drew attention to this issue as one of considerable importance for general equilibrium theory by providing an alternative approach to the same result. Other assumptions also were relaxed, but the lecture primarily was a major contribution to the survival issue. Of course, as in the 1959 contribution, the economy as a whole must be assumed to be able to survive.

This contribution makes McKenzie's concept of irreducibility of primary importance in the study of famines. As Sen (1981) has noted, "Starvation is a matter of some people not having enough food to eat, and not a matter of there being not enough food to eat." That is, the typical situation in a famine is that the economy as a whole can survive (by, for example, a "command system"), and it is a failure of the market mechanism that not all agents can survive. This market failure can then be attributed to a failure of irreducibility.

### 1.3 Summary

As students of McKenzie, we have shared a common experience. When we teach *our* students aspects of equilibrium, trade, or growth and encounter difficult terrain, we are led to look at original papers by McKenzie to see how he dealt with it. Invariably, we find the issue has been addressed by him, and more often than not a general way of overcoming the difficulty has been suggested. It is clear to us that in developing a theory, McKenzie aimed at a complete understanding of what makes such a theory work. With that aim, over roughly a forty year period, he made contributions of extraordinary depth on a wide range of important questions in economic theory.

In putting together this selection of his papers, we were principally guided by two objectives. First, we firmly believe that there is no better way to perceive the development of McKenzie's ideas than to look at the original papers in which they appeared together, not in isolation. Second, in contrast to the simplified versions of his contributions that have been readily available for years, the many state-of-the-art results achieved by McKenzie in the original papers present a much more sophisticated theory, which will continue to be important in future developments of the discipline.

## Notes

1. Our exposition of McKenzie's 1954 paper, and indeed of his subsequent papers on the existence of competitive equilibrium, is deliberately brief. In a book of this size and scope, we would not be able to do justice to the technical aspects, which are an integral part of the subject matter. A comprehensive discussion of McKenzie's four papers on the subject (published in 1954, 1956, 1959, and 1981) can be found in the paper by Ali Khan (1993).
2. This is the notion of equilibrium examined by von Neumann in the paper, previously mentioned in our discussion of McKenzie's contributions to the theory of existence of a competitive equilibrium. In that context it was primarily of interest for the novelty of the methods introduced, since it provided a result, which was effectively a generalization of Brouwer's fixed point theorem before Kakutani's well-known contribution on the same topic. In the turnpike theory context, von Neumann's concept of equilibrium itself was shown to be of interest by the conjecture of DOSSO, a point made most clearly in the expository piece by Koopmans (1964).
3. The demonstration of this result was sketched in DOSSO, but it was incomplete and restricted to two goods. Further, in its demonstration, although clearly not in its formulation, it was a "local result"; this local result was formally demonstrated in its full generality later by McKenzie in a paper published in the *International Economic Review* in 1963.
4. This was published later in a paper in *Econometrica* in 1963.
5. This was later published in a conference volume of the International Economic Association, titled "Activity Analysis in the Theory of Growth and Planning" in 1967.
6. This was first undertaken in the paper published in *Econometrica* in 1963, already mentioned.
7. This was published in a special issue in 1971 of the *Journal of Economics*, devoted to "Contributions to the von Neumann Growth Model."
8. His counterexample is contained in a manuscript written in 1962 but never published. It was reproduced in the paper by Gale and Nikaido in their 1965 paper in *Mathematische Annalen*.
9. Their famous global univalence result imposed the condition that the relevant Jacobian matrix have all its principal minors positive in a domain, which was taken to be rectangular, and has come to be known as the "P-matrix" condition.
10. This was published in the *International Economic Review* in 1967.
11. See, in particular, Mas-Collel's (1979) contribution to the first McKenzie *festschrift* for an account of the developments for the issue of global inversion of cost functions.
12. A systematic study of the nature of such preferences over an infinite horizon was initiated by Koopmans in 1960. Becker and Boyd (1998) provide a comprehensive account of the literature that since has developed on this theme.
13. Interest in studying the themes of Ramsey's paper had surfaced before, most notably in the research of Tinbergen (1956) and Chakravarty (1962), but it was not shared more broadly by the profession at that time.
14. See the *Review of Economic Studies* symposium of 1967, mentioned earlier.

15. See his beautiful and definitive contribution on the existence problem in the *Review of Economic Studies* (1970).
16. Atsumi's contributions were part of his doctoral thesis at the University of Rochester, under McKenzie's supervision, and were published in the *Review of Economic Studies* in 1965 and 1969.
17. Equivalently, one writes the transition possibility set as transforming a vector of beginning-of-period stocks of goods to a vector consisting of end-of-period stocks of goods and a "utility good." It is this formulation that appears in McKenzie's contribution to the Hicks *festschrift*. The "reduced form utility function" would then be obtained by fixing a pair of (feasible) beginning-of-period and end-of-period stocks of goods, and maximizing the component of the "utility good."

## References

- Arrow, K. J., and G. Debreu. (1954). Existence of an Equilibrium for a Competitive Economy. *Econometrica* 22, 265–290.
- Arrow, K. J., and F. H. Hahn. (1971). *General Competitive Analysis*. Holden-Day: San Francisco.
- Atsumi, H. (1965). Neoclassical Growth and the Efficient Program of Capital Accumulation. *Rev. Econ. Studies* 32, 127–136.
- Atsumi, H. (1969). The Efficient Capital Program for a Maintainable Utility Level. *Rev. Econ. Studies* 36, 263–288.
- Becker, R., and J. Boyd. (1997). *Capital Theory, Equilibrium Analysis, and Recursive Utility*. Basil Blackwell: Oxford.
- Brock, W. A. (1970). On Existence of Weakly Maximal Programmes in a Multi-sector Economy. *Review of Economic Studies* 37, 275–280.
- Brock, W. A., and J. Scheinkman. (1976). Global Asymptotic Stability of Optimal Control Systems with Applications to the Theory of Economic Growth. *J. Econ. Theory* 12, 164–190.
- Brock, W. A., and J. Scheinkman. (1978). On the Long-Run Behavior of a Competitive Firm, in G. Schwodiauer (ed.), *Equilibrium and Disequilibrium in Economic Theory*. Dordrecht: D. Reidel.
- Cass, D., and K. Shell. (1976). The Structure and Stability of Competitive Dynamical Systems. *J. Econ. Theory* 12, 31–70.
- Chakravarty, S. (1962). The Existence of an Optimum Savings Programme. *Econometrica* 30, 178–187.
- Debreu, G. (1952). A Social Equilibrium Existence Theorem. *Proc. Nat. Acad. Sci., USA* 38, 886–893.
- Debreu, G. (1959). *Theory of Value*. Wiley: New York.
- Debreu, G. (1970). Economies with a Finite Set of Equilibria. *Econometrica* 38, 387–392.
- Dorfman, R., P. Samuelson, and R. Solow. (1958). *Linear Programming and Economic Analysis*. McGraw-Hill: New York.



- Gale, D. (1955). The Law of Supply and Demand. *Math. Scand.* 30, 155–169.
- Gale, D. (1967). On Optimal Development in a Multi-Sector Economy. *Rev. Econ. Stud.* 34, 1–18.
- Gale, D., and H. Nikaido. (1965). The Jacobian Matrix and the Global Univalence of Mappings, *Math. Ann.* 159, 81–93.
- Hicks, J. R. (1939). *Value and Capital*. Oxford: Oxford University Press.
- Hicks, J. R. (1961). The Story of a Mare's Nest, *Rev. Econ. Stud.* 76, 77–88.
- Kakutani, S. (1941). A Generalization of Brouwer's Fixed Point Theorem, *Duke Math J.* 8, 457–459.
- Khan, M. Ali. (1993). Lionel McKenzie on the Existence of Competitive Equilibrium, in R. Becker, M. Boldrin, R. Jones, and W. Thomson (eds.), *General Equilibrium, Growth and Trade II*. Academic Press: New York.
- Koopmans, T. C. (1951). Analysis of Production as an Efficient Combination of Activities, in T. C. Koopmans (ed.), *Activity Analysis of Production and Distribution*. New York: Wiley.
- Koopmans, T. C. (1957). *Three Essays on the State of Economic Science*. McGraw-Hill.
- Koopmans, T. C. (1960). Stationary Ordinal Utility and Impatience, *Econometrica* 28, 287–309.
- Koopmans, T. C. (1964). Economic Growth at a Maximal Rate. *Quart. Journ. Econ.* 78, 355–394.
- Kurz, M. (1968). Optimal Economic Growth and Wealth Effects. *Int. Econ. Rev.* 9, 155–174.
- Mas-Colell, A. (1979). Two Propositions on the Global Univalence of Systems of Cost Function, in J. Green and J. A. Scheinkman (eds.), *General Equilibrium, Growth, and Trade*. Academic Press: New York.
- McFadden, D. (1967). The Evaluation of Development Programmes. *Rev. Econ. Studies* 34, 25–50.
- McKenzie, L. W. (1951). Ideal Output and the Interdependence of Firms, *Economic Journal*, 785–803.
- McKenzie, L. W. (1954). On Equilibrium in Graham's Model of World Trade and Other Competitive Systems, *Econometrica*, 147–161.
- McKenzie, L. W. (1954). Specialization and Efficiency of World Production, *Review of Economic Studies*, 165–180.
- McKenzie, L. W. (1955). Equality of Factor Prices in World Trade, *Econometrica*, 239–257.
- McKenzie, L. W. (1955). Specialization in Production and the Production Possibility Locus, *Review of Economic Studies*, XXIII (1), 56–64.
- McKenzie, L. W. (1956). Competitive Equilibrium with Dependent Consumer Preferences, in H. A. Antosiewicz (ed.), *Second Symposium in Linear Programming*, vol. I. National Bureau of Standards, Washington, DC, 277–294.

- McKenzie, L. W. (1957). Demand Theory without a Utility Index, *Review of Economic Studies*, XXIV (3), 185–189.
- McKenzie, L. W. (1957). An Elementary Analysis of the Leontief System, *Econometrica*, 456–462.
- McKenzie, L. W. (1959). On the Existence of General Equilibrium for a Competitive Market, *Econometrica*, 54–71.
- McKenzie, L. W. (1959). Matrices with Dominant Diagonals and Economic Theory, in K. J. Arrow, S. Karlin, and P. Suppes (eds.), *Mathematical Methods in the Social Sciences*. Stanford University Press, 47–62.
- McKenzie, L. W. (1960). Stability of Equilibrium and the Value of Excess Demand, *Econometrica*, 606–617.
- McKenzie, L. W. (1961). On the Existence of General Equilibrium: Some Corrections, *Econometrica*, 247–248.
- McKenzie, L. W. (1963). The Dorfman-Samuelson-Solow Turnpike Theorem, *International Economic Review*, 29–43.
- McKenzie, L. W. (1963). Turnpike Theorems for a Generalized Leontief Model, *Econometrica*, 165–180.
- McKenzie, L. W. (1963). Turnpike Theorem of Morishima, *Review of Economic Studies*, 169–176.
- McKenzie, L. W. (1967). Maximal Paths in the Von Neumann Model, in E. Malinvaud and M. O. L. Bacharach (eds.), *Activity Analysis in the Theory of Growth and Planning*. Macmillan: London, 43–63.
- McKenzie, L. W. (1967). The Inversion of Cost Functions: A Counter-Example, *International Economic Review*, 271–278.
- McKenzie, L. W. (1967). Theorem and Counter Example, *International Economic Review*, 279–285.
- McKenzie, L. W. (1968). Accumulation Programs of Maximum Utility and the Von Neumann Facet, in J. N. Wolfe (ed.), *Value, Capital, and Growth*. Edinburgh University Press, 353–383.
- McKenzie, L. W. (1971). Capital Accumulation Optimal in the Final State, in G. Bruckmann and W. Weber (eds.), *Contributions to the Von Neumann Growth Model*. Springer-Verlag: New York, 107–120.
- McKenzie, L. W. (1974). Turnpike Theorems with Technology and Welfare Function Variable, in J. Los and M. Los (eds.), *Mathematical Models in Economics*. North Holland: Amsterdam, 271–288.
- McKenzie, L. W. (1976). Why Compute Competitive Equilibria? in J. Los and M. Los (eds.), *Computing Equilibria: How and Why*. North Holland: Amsterdam, 3–19.
- McKenzie, L. W. (1977). Turnpike Theory, *Econometrica*, 841–865.
- McKenzie, L. W. (1977). A New Route to the Turnpike, in R. Henn and O. Moeschlin (eds.), *Mathematical Economics and Game Theory*. Springer-Verlag: New York, 683–694.

McKenzie, L. W. (1981). The Classical Theorem on Existence of Competitive Equilibrium, presidential address to the Econometric Society, *Econometrica*, 819–841.

McKenzie, L. W. (1982). A Primal Route to the Turnpike and Liapounov Stability, *Journal of Economic Theory* 27, 194–209.

McKenzie, L. W. (1983). Turnpike Theory, Discounted Utility, and the Von Neumann Facet, *Journal of Economic Theory* 30, 330–352.

McKenzie, L. W. (1986). Optimal Economic Growth, Turnpike Theorems and Comparative Dynamics, in K. J. Arrow and M. D. Intriligator (eds.), *Handbook of Mathematical Economics*, vol. III, North Holland: Amsterdam, 1281–1355.

McKenzie, L. W. and J. H. Boyd III. (1993). The Existence of Competitive Equilibrium over an Infinite Horizon with Production and General Consumption Sets, *International Economic Review* 34, 1–20.

McKenzie, L. W. (1999). A Scholar's Progress, *Keio Economic Studies* 36, 1–12.

McKenzie, L. W. (1999). Equilibrium, Trade, and Capital Accumulation, *The Japanese Economic Review* 50, 371–397.

McKenzie, L. W. (2002). Some Early Conferences on Growth Theory, in G. Bitros and Y. Katsoulacos (eds.), *Essays in Economic Theory, Growth, and Labor Markets*. Edward Elgar: Cheltenham, UK.

Moore, J. (1975). The Existence of Compensated Equilibrium and the Structure of the Pareto Efficiency Frontier, *Int. Econ. Rev.* 16, 267–300.

Morishima, M. (1961). Proof of a Turnpike Theorem: The No Joint Production Case, *Rev. Econ. Studies* 28, 89–97.

Nash, J. F. (1950). Equilibrium Points in N-Person Games, *Proc. Nat. Acad. Sci., USA* 36, 48–59.

Negishi, T. (1960). Welfare Economics and Existence of Equilibrium for a Competitive Economy, *Metroeconomica* 12, 92–97.

Nikaido, H. (1956). On the Classical Multilateral Exchange Problem. *Metroeconomica* 8, 135–145.

Nikaido, H. (1968). *Convex Structures and Economic Theory*. Academic Press: New York.

Pearce, I. (1959). A Further Note on Factor Commodity Price Relationships, *Econ. J.* 69, 725–732.

Pearce, I. F., and S. F. James. (1952). The Factor Price Equalization Myth, *Rev. Econ. Stud.* 19, 111–120.

Radner, R. (1961). Paths of Economic Growth That Are Optimal with Regard Only to Final States. *Rev. Econ. Stud.* 28, 98–104.

Ramsey, F. P. (1928). A Mathematical Theory of Saving. *Econ. J.* 38, 543–559.

Rockafellar, R. T. (1976). Saddle Points of Hamiltonian Systems in Convex Lagrange Problems Having a Non-Zero Discount Rate. *J. Econ. Theory* 12, 71–113.

Samuelson, P. A. (1953). Prices of Factors and Goods in General Equilibrium, *Rev. Econ. Stud.* 21, 1–20.

- Samuelson, P. A. (1973). Optimality of Profit-Including Prices under Ideal Planning, *Proc. Nat. Acad. Sci., USA* 70, 2109–2111.
- Scheinkman, J. (1976). On Optimal Steady State of N-Sector Growth Models When Utility Is Discounted, *J. Econ. Theory* 12, 11–30.
- Sen, A. (1981). Ingredients of Famine Analysis: Availability and Entitlements, *Quarterly Journal of Economics* 96, 433–464.
- Sutherland, W. (1970). On Optimal Development in a Multi-sectoral Economy: The Discounted Case, *Rev. Econ. Studies* 37, 585–589.
- Tinbergen, J. (1956). The Optimum Rate of Saving, *Econ. J.* 66, 603–609.
- von Neumann, J. (1945). A Model of General Economic Equilibrium, *Rev. Econ. Stud.* 13, 1–9 (translated from the original German paper of 1935–1936 by G. Morton).
- Wald, A. (1951). On Some Systems of Equations of Mathematical Economics, *Econometrica* 19, 368–403 (translated from the original German paper of 1936 by O. Eckstein).
- Weintraub, E. R. (1983). The Existence of a Competitive Equilibrium: 1930–1954, *Journal of Economic Literature*, 1–39.
- Weintraub, E. R. (1985). *General Equilibrium Analysis*. Cambridge University Press.